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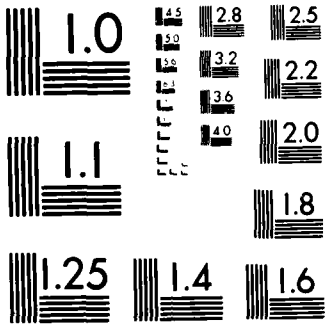

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Electron Beam Trajectory in a Photometer Field of View

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16 February 1983

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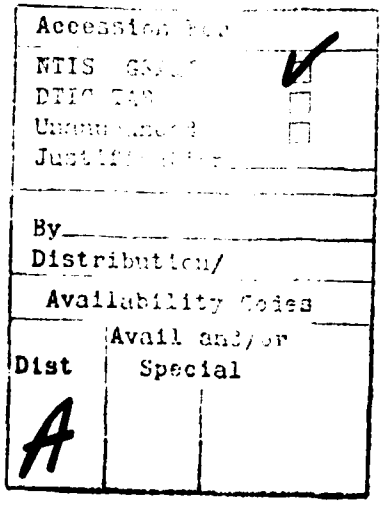


Electron Beam Trajectory in a Photometer Field of View

I. INTRODUCTION

From the inception of the use of electron beams on sounding rocket flights, photometers have been used to measure the light produced by the interaction of the electron beams with the gas surrounding the rocket payload.[1,2] Since the path of an electron is affected by the earth's magnetic field, the measurable, the luminosity of the beam-gas interaction in the field-of-view of the photometer is also similarly affected. Early experiments tried to minimize these effects by making measurements close to the payload and with a restricted field-of-view for the photometer. For measurements away from the vehicle, and for wide viewing angles, explicit calculations of particle trajectories in the field-of-view of optical devices are necessary for the planning and interpretation of experiments.

[illegible] for publication 14 February 1972

1. [illegible], R., and Davies, W.E.R. (1970) Spatial Rocket Instrumentation [illegible], North Holland Publishing Co., Amsterdam, Holland.

2. [illegible], R., and Davies, W.E.R. (1971) Measurements of [illegible] [illegible], Can. J. [illegible]

2. GEOMETRY OF INSTRUMENTATION

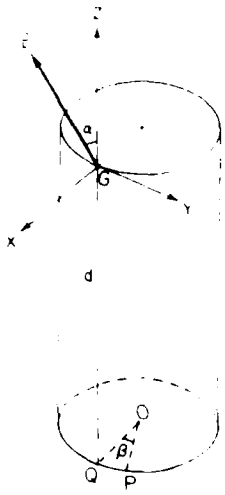


Figure 1. Geometry of Electron Gun G and Photometer P Mounted on a Section of the Rocket Body. The origin of the coordinate system is at G, the z-distance from the gun to the photometer is d

In the x, y, z coordinate system shown in Figure 1, the gun G is located at the origin. The photometer P is mounted at $-\rho + \rho \cos \beta$, $\rho \sin \beta$, -d. At any time t, let a beam electron be located at x, y, z. It subtends angles θ_x and θ_y at the photometer:

$$\theta_x = \tan^{-1}\left(\frac{x + \rho - \rho \cos \beta}{z + d}\right), \tag{1}$$

$$\theta_y = \tan\left(\frac{y - \rho \sin \beta}{z + d}\right) . \tag{2}$$

In the special case $\beta = 0$, the photometer would be located directly below G at distance d, (see Figure 2).

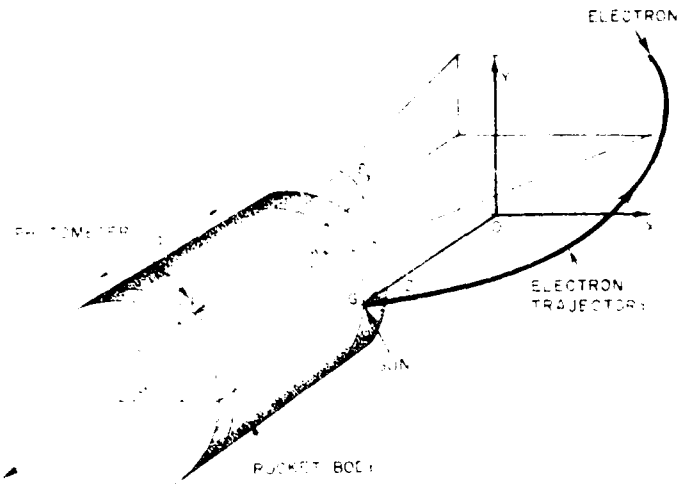


Figure 2. Geometry of Electron Gun G and Photometer P at Any Instant of Time. Beam electron subtends angles θ_x and θ_y at Photometer P

The field-of-view of the photometer is designed to be limited. It is assumed that the field is circular with the center at $\theta_x(c)$ and $\theta_y(c)$ and radius θ_r. An electron at $\theta_x(t)$, $\theta_y(t)$, would be in the photometer field-of-view if

$$[\theta_x(t) - \theta_x(c)]^2 + [\theta_y(t) - \theta_y(c)]^2 \leq \theta_r^2 . \qquad (6)$$

If the above inequality is not satisfied, the electron is outside the photometer field-of-view (see Figure 3).

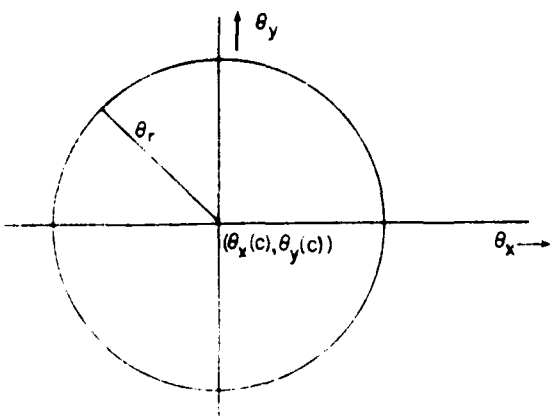


Figure 3. Circular Angular Field-of-View of Photometer

3. ELECTRON TRAJECTORY–B COORDINATE SYSTEM

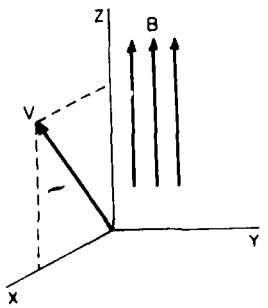
The luminosity of the beam atmosphere interaction measured in the photometer's field-of-view is sensitive to the magnetic field orientation with respect to the beam or rocket. Part of the electron trajectory may move in or out of the field-of-view.

In the B system of coordinates as defined in Figure 4, the magnetic field $\vec{B}$ is parallel to the z axis. The vector $\vec{V}$ is the initial beam velocity. The v axis is defined as along $\vec{z} \times \vec{V}$. Thus, $\vec{V}$ lies in the z-x plane. The origin of the beam is at x = 0, y = 0, z = 0. The equation of motion of a beam in the B system is as follows:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_B = \begin{bmatrix} R \sin \omega t \\ R - R \cos \omega t \\ V_{||} t \end{bmatrix} \tag{4}$$

where

$$\omega = \frac{eB}{mc}$$



B — System

Figure 4. B-Coordinate System. Magnetic field $\vec{B}$ is defined along z-axis

and

$$R \quad \frac{V_{\perp}}{\omega}$$

where R is gyroradius, ω is gyrofrequency, t is time, and $V_{\parallel}$ and $V_{\perp}$ are the velocity component parallel and perpendicular to the magnetic field respectively. The energy E of the electron is related to the velocity V by $E = 1/2\ M V^2$, where m is the mass of electron.

In the B system, the $\vec{B}$ vector is fixed, the $\vec{V}$ vector varies but lies in the z-x plane, and, as the rocket spins, the photometer look-angle varies with time.

4. ELECTRON TRAJECTORY–R COORDINATE SYSTEM

In order to study the electron trajectory in the field-of-view of the photometer, it is more convenient to define an R system of coordinates in which the photometer look-angle is always fixed and the $\vec{B}$ vector varies with time. In the R system (see Figure 5), the z axis is defined as parallel to the rocket axis, y axis is in radial direction, and x completes the right-handed system.

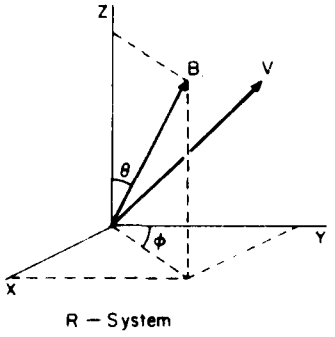


Figure 5. R Coordinate System. The z-axis is parallel to rocket body axis. This is the same coordinate system used in Figure 1.

At time t, let the magnetic field vector $\vec{B}$ be in arbitrary direction, defined by pitch angle $\theta(t)$ and azimuth angle $\phi(t)$ in the R system (see Figure 5).

The equation of motion of a beam electron is obtained in the R system by an orthogonal transformation from Eq. (4) in the B system.

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_R \qquad \begin{bmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \qquad \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_B \tag{5}$$

where (ℓ_i, m_i, n_i) are the direction cosines of the i-th basis vector of the R system with the B system.

In terms of cross products, the transformation equation is as follows:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_R \qquad \begin{bmatrix} (\vec{B} \cdot \vec{V}) \cdot \vec{B} & (\vec{B} \cdot \vec{V}) \cdot \vec{B} \\ \vec{B} \cdot \vec{V} & \vec{B} \cdot \vec{V} \\ \vec{B} & \vec{B} \end{bmatrix}^{-1} \qquad \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_B \quad . \tag{6}$$

In the special case when the magnetic field $\vec{B}$ lies in the plane of the initial electron beam direction and the rocket axis, the transformation equation becomes

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_R = \begin{bmatrix} 0 & 1 & 0 \\ -\cos\theta & 0 & \sin\theta \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}_B \tag{7}$$

where

$$\frac{\vec{B}}{B} = (0, \cos\theta, \sin\theta) \, .$$

5. LUMINOSITY

The angular coordinates θ_x and θ_y of an electron as viewed by the photometer are given by Eqs. (1) and (2).

Using the orthogonal transformation equation, Eq. (4), one can write down the angular coordinates (Figure 2), as viewed by a photometer lying at (0, 0, -d):

$$\begin{aligned} \theta_x(t) &= \tan^{-1}\left(\frac{x(t) + \rho - \rho\cos\beta}{z(t) + d}\right) \\ \theta_y(t) &= \tan^{-1}\left(\frac{y(t) - \rho\sin\beta}{z(t) + d}\right) \quad . \end{aligned} \tag{8}$$

The electron would lie in the photometer field-of-view if Eq. (3) is satisfied (see Figure 3). Equivalently, let us define a function F(t):

$$F(t) = \frac{r_2^2}{2} - \left\{ [\theta_x(t) - \theta_x(c)]^2 + [\theta_y(t) - \theta_y(c)]^2 \right\} .$$

If Eq. (3) is satisfied, then the function F(t) is positive, (see Figures 6 and 7),

$$F(t) > 0 , \tag{9}$$

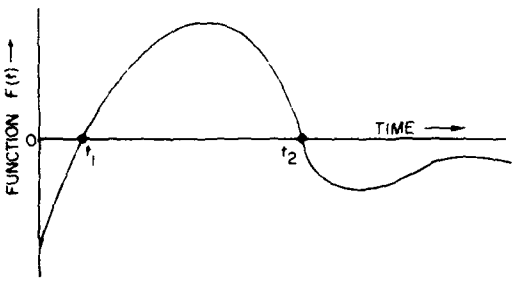


Figure 6. Typical Behavior of Function F(t). The beam electron is in the photometer field-of-view during the period t_1 to t_2

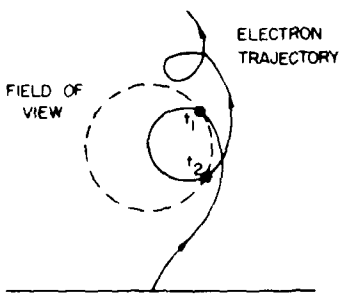


Figure 7. Electron Beam Trajectory in Photometer Field-of-View. The beam entering field-of-view (dashed circle) at t_1 and leaves at t_2

[illegible] with a gas atom, or molecule, to create ionization or [illegible], the luminosity I(E) measured at the photometer is proportional to the [illegible] of the distance s(t) of the electron from the photometer.

$$\propto \int \frac{1}{\frac{1}{2}|F(t)|} \tag{10}$$

$$\propto \int \frac{1}{\frac{1}{2}|F(t)|}$$

where N is a proportionality constant, V is the velocity of the electron, and $\sigma(E)$ is the cross-section of ionization and excitation. It is assumed that no significant energy loss ($\Delta E/E << 1$) takes place along the beam while in the photometer field-of-view. $\textcircled{H}(x)$ is a step function:

$$\textcircled{H}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} .$$

From Eq. (10), one obtains the geometric factor G as follows:

$$L(E) = N V(E) \sigma(E) G$$

where

$$G = \sum_{i=1}^{\infty} \int_{t_{2i-1}}^{t_{2i}} dt \, \frac{1}{s^2(t)}$$

$$s(t) = \left[x^2(t) + v^2(t) + (z + d)^2\right]^{1/2} .$$

When the initial velocity vector $\vec{V}$ of the electron is perpendicular to the magnetic field vector $\vec{B}$, the beam forms a circular path. The condition is

$$\vec{B} \cdot \vec{V} = 0 . \qquad (11)$$

Every electron injected into the path would stay in the path and never propagate away (see Figure 8). This is a nonpropagation mode. The luminosity I(E) for this mode is high, because $s^{-2}(t)$ does not decrease with time.

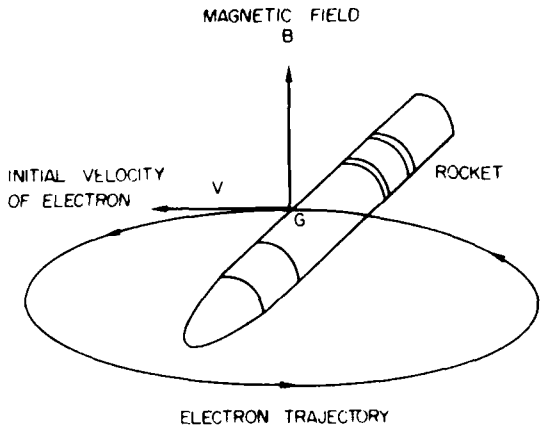


Figure 8. Circular Electron Trajectory When Initial Electron Velocity is Perpendicular to the Magnetic Field

The above condition agrees with the experimental result of Israelson and Winckler[3] who detected a substantial increase in photon flux at 90° pitch angle in the Echo 2 rocket beam experiment.

6. SCEX ROCKET

As an example, for the SCEX rocket* experiment the specifications of the photometer on-board is given in Table 1.

Table 1. SCEX Photometer Specifications

Photometer rotation angle β	= 0°
Photometer distance from gun (d)	= 119 cm
Center of field-of-view [θ_x(c), θ_y(c)]	= [0°, 20°]
Radius of field-of-view θ_r	= 15°

The condition of Eq. (11) for nonpropagation modes to exist becomes

$$\sin\theta\cos\phi + \cos\theta = 0 \tag{12}$$

where θ and ϕ are rocket pitch and azimuth angles. The solutions of Eq. (12) are plotted in Figure 9.

In Figure 9, the pitch angle actually runs only from 0° to 180° because it is a cone angle. The azimuth angle ϕ at 180° is the same as -180° because it is a rotation angle. Solutions exist only for a range of values of pitch angle.

Examples of electron beam trajectories as viewed at the SCEX photometer are presented in Figures 10 and 11. Three-dimensional plots of luminosity, for the case of SCEX, as a function of θ and ϕ, are shown in Figures 12 and 13. The location of the spikes should fall on a continuous curve given in Eq. (12) (Figure 9) but computer calculation requires the use of grid points which are discrete. Therefore the singularities do not look like a continuous wall, but appear as spikes. The 1900-eV case gives a generally higher (about 2 to 3 times) luminosity than the

3. Israelson, G., and Winckler, J.R. (1975) Measurement of 3914-A light production and electron scattering from electron beams artificially injected into the ionosphere, J. Geophys. Res. 80:No. 25:3709-3712.

*NASA Rocket 27.045, launched on 27 January 1982, from Churchill, Canada.

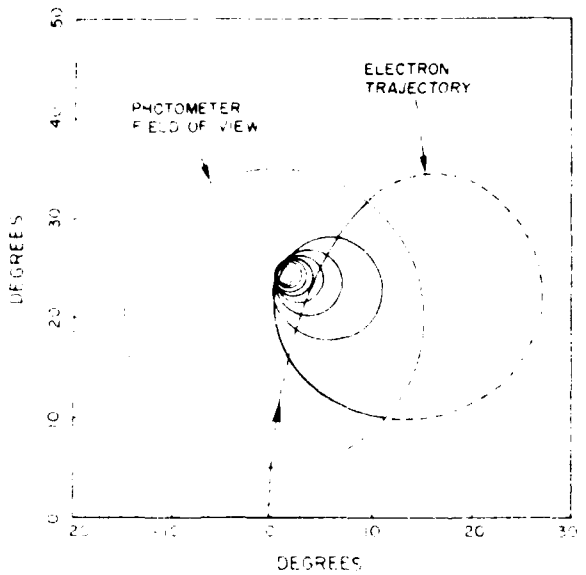
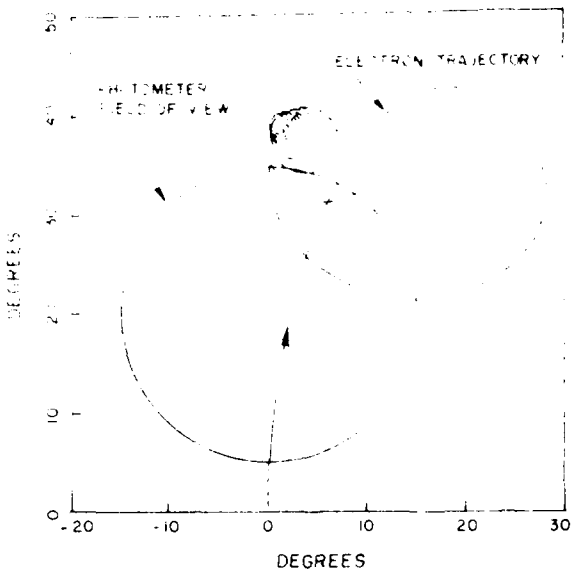


Figure 10. Computer Simulation of Electron Beam Trajectory as Viewed at the Photometer on the SCEX Rocket, the Magnetic Field Look-angle in This Simulation is (0°, 25°). The dashed part of the trajectory is outside the circular field of view.



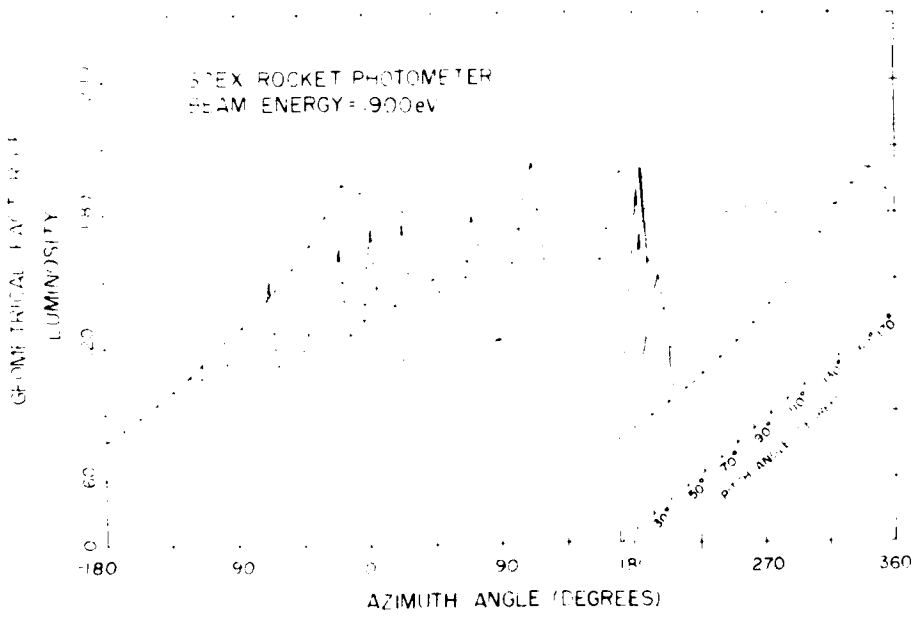


Figure 12. Geometrical Factor of Luminosity of the Electron Beam as Viewed by the Photometer on the SCEX Rocket. Beam Energy is 1900 eV. The functional dependence on pitch and azimuth angles are plotted

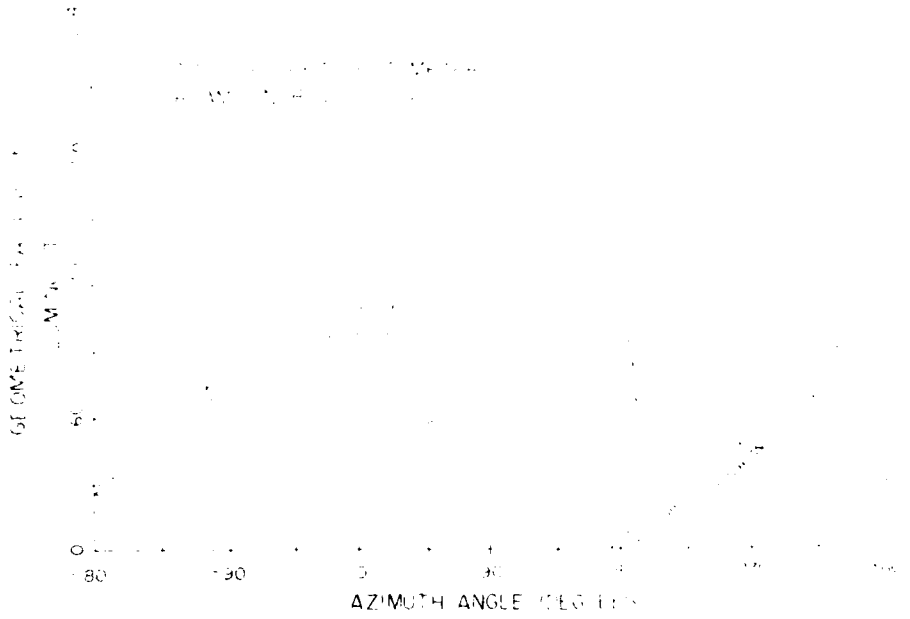
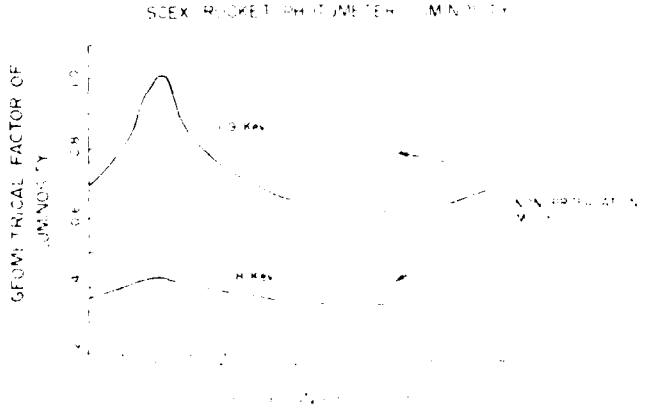


Figure 13. Geometrical Factor of Luminosity of the Electron Beam as Viewed by the Photometer on the SCEX Rocket. Beam Energy is 8000 eV. The functional dependence on pitch and azimuth angles are plotted



Appendix A

Blockage of Field-of-View by the Horizon

The blockage of the field-of-view, for a wide angle photometer P, is determined by the horizon. For the geometry shown in Figure A1, the equation of the horizon is

$$y = mx + c ,$$

where

$$m = -\tan \beta$$

$$c = \rho(\sec \beta - 1) .$$

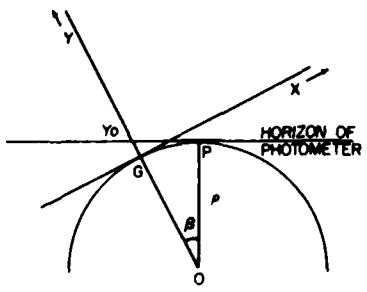


Figure A1. Blockage of Photometer Field-of-View by the Horizon

If at x = 0, blockage is desired to be below y_0, then

$$\rho(\sec\beta - 1) \leq y_0 ,$$

that is

$$\frac{1}{\cos\beta} \leq \frac{y_0 + \rho}{\rho} .$$

This determines the maximum β, (see Figure A2).

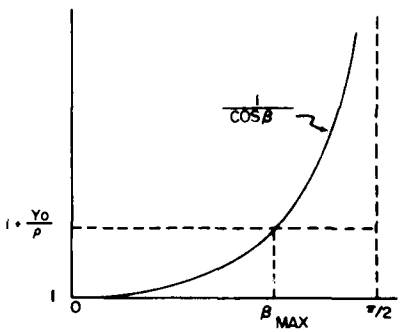
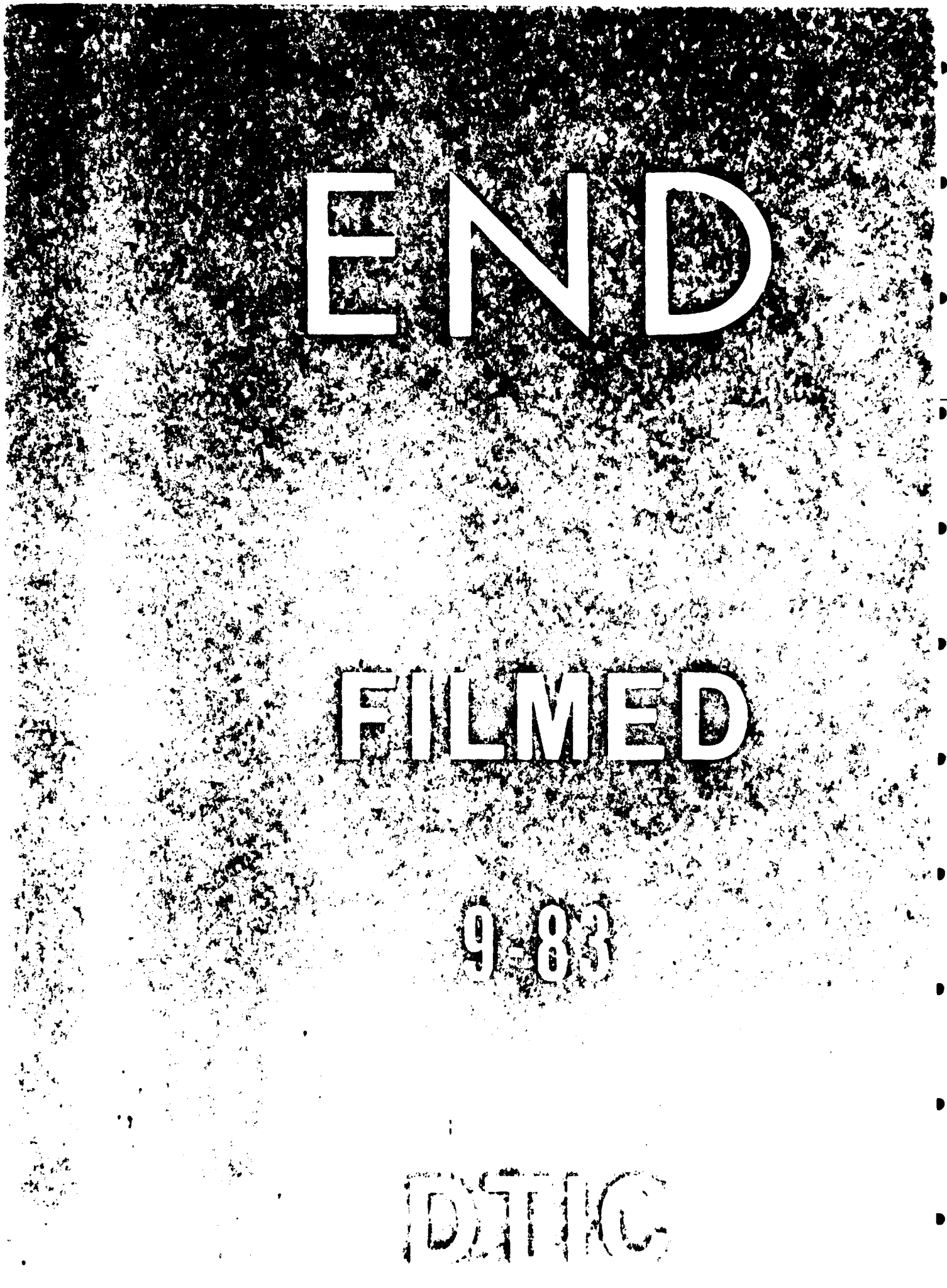


Figure A2. Solution of the Maximum Angle β_{max}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